Restructuring School Physics around Real-World Problems: A Cognitive Justification

David Fortus

Michigan State University, East Lansing, MI

Paper presented at the annual meeting of the American Educational Research Association

April 14, 2005, Montreal, Canada

David Fortus
Michigan State University
330 Erickson Hall
East Lansing, MI 48824-1034
USA
Tel: (517)-432-9925
Fax: (517)-432-1502
dfortus@msu.edu

This material is based on work supported by the National Science Foundation under Grant No. ESI-0227557. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Well- and Ill-Defined Problems

There are many things, such as using an electron microscope, playing the cello or reading Egyptian hieroglyphs, which most of us will never learn to do, unless we spend time deliberately learning how to do them. Everyday problem-solving is not one of these things. Every time we go shopping and decide which shop we'll visit first, which second and which third, we are strategizing. When a certain person never answers our emails, we hypothesize what may be the reason for that person's lack of response. When our airplane waits for an hour in a queue before it can takeoff, we may generalize that flights always leave late from that airport. We all engage in problem-solving activities such as hypothesizing, strategizing, generalizing, and inferring, without the benefit of instruction, because we must do so in order to handle the challenges of daily life (Nickerson, 1994).

But just because we naturally engage in problem-solving doesn't mean that we are necessarily good at it or that we couldn't get better. Our reasoning is often biased in ways that prevent us from reaching satisfactory solutions to the problems facing us (Evans, 1989). Many students are unable to engage in the problem solving activities that their school's curricular programs require (International Association for the Evaluation of Educational Achievement. & TIMSS International Study Center., 1996; Lapointe *et al.*, 1989).

Problems can be divided into two categories: well-defined and ill-defined. Well-defined problems are those for which there are well-defined starting and ending states, for which it is theoretically possible to describe all the possible steps that can be made in solving the problem. These problems have a distinct solution-space. There is a "legal move generator" for finding all the possibilities at each step, and from them to pick the sequence of steps that provide the best or most efficient solution to the problem (Newell & Simon, 1972; Simon, 1978). Ill-defined problems, on the other hand, are residuals; they are defined in terms of what they are not – they are all those problems that are not well-defined (Simon, 1973). The criteria that determine whether an ill-defined problem's solution has been reached may be either lacking or indefinite; indeed, it may not have a well-defined solution. Often the information required to solve ill-defined problems is missing or vague, leading to uncertainty regarding the operations and the materials that can be used in the solution (Glass & Holyoak, 1986; Newell, 1969; Reitman, 1964; Simon, 1978).

Not all ill-defined problems are equally ill-defined. Some may be almost well-defined. Consider the problem of baking a cake. One way to solve this problem is by following a recipe in a cookbook. However, even though the exact amount of each required ingredient is specified by the cookbook, it leaves open the issue of which brands of ingredients to buy and where to purchase them. Other problems may be almost completely ill-defined, such as the problem of how to compose a symphony. Rather than characterize ill-defined problems as residuals, Reitman (1964) thought it more useful to view all problems as being located on a one-dimensional continuum, with well-defined problems at one end and highly ill-defined problems at the other end. I prefer to expand this perspective, and to position all problems on an N-dimensional continuum, where each dimension represents a particular aspect of the problem, such as the definition of the goal state, availability of information needed to solve the problem, and so forth. Well-defined problems such as tic-tac-toe will be located near one 'corner' of the continuum and such highly ill-defined problems as composing a symphony will be near the opposing 'corner'.

Most of the problems that we face in our everyday lives are ill-defined to some degree (Frederiksen, 1986; Glass *et al.*, 1979; Nickerson, 1994; Reitman, 1964; Roberts, 1995). For example, what to do for your child's birthday party, what to cook for supper, and how to ensure that you will have a continuous and plentiful supply of drinking water are all, in varying degrees, ill-defined problems.

Finding a satisfactory solution to a problem involves searching through a solution space (Newell & Simon, 1972). As comparing tic-tac-toe to chess clearly demonstrates, the time required to find an acceptable solution grows rapidly with increasing size of the solution space; therefore, limiting the size of the solution space by reformulating the problem or by adding constraints can greatly assist in solving a problem.

A main difference between well-defined and ill-defined problems is the degree to which they are constrained. A problem's constraints can determine what information and materials are pertinent to the problem, and what operations are allowed in solving the problem. A standard technique used to solve ill-defined problems is to apply to it additional subjectively chosen constraints, thereby making it 'more' defined and limiting the size of its solution space (Reitman, 1964). We would therefore expect that experts at the solution of ill-defined problems in a particular domain, such as Newtonian mechanics, should also be good at solving well-defined problems in the same domain. The opposite, however, need not be true. People who are proficient at solving well-defined problems in a particular domain may not be good at solving ill-defined problems in the same domain since they may not be skillful at making these problems 'more' defined.

School science has been traditionally built around well-defined problems, such as predicting an ideal projectile's trajectory or calculating how much hydrogen is released by the dissociation of a given amount of water. On the other hand, real-world scientific inquiry focuses on ill-defined problems: "There simply is no fixed set of steps that scientists always follow, no one path that leads them unerringly to scientific knowledge" (AAAS, 1990, p. 4).

If indeed being skillful at solving ill-defined problems in a particular domain means being skillful at solving well-defined problems in the same domain as well, but not vice-versa, then the calls to restructure school science around real-world problems receive additional supported, since traditional science curricula that are based on well-defined problems do not give students experience in choosing, justifying, and applying subjective constraints, skills which are necessary in order to cope successfully with real-world problems.

This study attempts to answer the following questions: "*Does skill at solving well-defined problems in Newtonian mechanics necessarily lead to skill at solving well-defined problems in the same domain? In which ways do the strategies used to solve well-defined physics problems differ from those used to solve ill-defined problem in the same domain?*"

Methods

Participants

There were eight participants from in the study. All eight were faculty, post-doctoral fellows, or graduate students at major research institutes. Participant A was both a professor of physics and a professor of science education, with over thirty years experience in both fields. Before obtaining his PhD in physics he had taught high school physics. Participant B was an assistant professor of physic with two years experience teaching introductory physics courses. Participant C was a post-doctoral fellow in science education with a PhD in science education and a BA in physics, with 1 year experience teaching physics at a high school and 5 years at the undergraduate level. Participants D – G were science education graduate students with undergraduate majors in physics. All but participant G had taught high school physics for several of years. All the participants stated that their entire secondary and undergraduate physics education had been based on well-defined problems. As teachers, only participant E had attempted to include ill-defined physics problems in their curricula. Thus it can be said that only participants A and B had any regular exposure to ill-defined physics problems, and this exposure was in the context of their research, not their teaching.

Instruments

All five participants were individually presented with the following four problems in Newtonian mechanics:

1. A person is pouring buckets of water into a cylindrical swimming pool with a diameter of 15m. In an hour, the water level of the pool rises by 2cm. Assuming the person fills the 10-liter bucket and pours its contents into the swimming pool at a constant rate, how many buckets of water does the person pour into the pool every minute?
2. A ball with a mass of 800gr is dropped, hitting the ground with a speed of 1.0m/s. The ball rebounds at a speed of 0.8m/s. The impact lasted 1/20th of a second. What was the average net force the ball was subjected to during the impact?
3. A block is pushed in order to start it sliding down an inclined plane with an angle of 30˚. The coefficient of friction between the plane and the block is velocity dependent: it is equal to $\mu_k = 0.15v$, where v is the block's speed in m/s relative to the plane. Assuming the plane is very long, what is the maximum speed relative to the plane that the block can attain?
4. A company thinks that there is a market for ultra-light umbrellas. While developing these umbrellas, the design team was confronted with the following question: Is the force of the rain falling on an opened umbrella a force that needs to be taken into consideration in designing an umbrella? You have been hired as a consultant to this firm. Your task is to estimate the magnitude of this force. You are allowed to use <u>anything</u> that you feel may assist you. You must be able to justify your solution before the members of the design team.

The first three of these problems were well-defined: no subjective assumptions were needed and all the information required to solve them that was unique to these problems was supplied. Each of these three questions covered a different topic in Newtonian mechanics: a) rate, total change, and time; b) linear momentum and impulse; and c) terminal velocity with velocity-dependent friction. The fourth question was ill-defined – it required the solver to make subjective assumptions and it lacked all the information required in order to reach a solution. Its solution required the application of the same concepts in Newtonian mechanics as the three well-defined questions. Thus, it could be reasonably assumed that anyone who was able to solve the ill-defined question should be able to solve the well-defined ones as well. The level of difficulty of the three well-defined questions was such that a high school AP physics graduate should be able to solve them without great difficulty. Had the assumptions and the information needed to solve the ill-defined problem been presented along with the problem, the same could have been said for this problem.

Each participant was encouraged to think-aloud (Ericsson & Simon, 1984) while solving the problems, first the three well-defined ones, and then the ill-defined one. The entire process was videotaped, with the camera focusing on the sheet of paper on which the participants wrote their solutions. No mention was made of the similarity of the underlying concepts between the first three and final problems. Before being taped, the participants were told only that the study was investigating how different people solve physics problems. After solving or attempting to solve all four problems, the participants were interviewed about the different difficulties the problems posed, where they struggled, whether they noticed any connection between the problems, the relative importance of the problems, and whether they believed their high school or undergraduate students would have been able to cope with them.

The appendix presents example solutions for the four problems. These solutions were developed by the author but resemble several of those developed by the participants.

Analysis

Each recording was analyzed in order to determine the various steps the participants engaged in while solving the problems. An initial framework for this analysis was taken from Bransford and Stein's IDEAL problem-solving model (1984), which includes the following steps: a) **I**dentify the problem, b) **D**efine and represent the problem, c) **E**xplore possible strategies, d) **A**ct on the strategies, and e) **L**ook back and evaluate the effects of your activities. As will be explained in the results section, this framework needed to be modified slightly to account for all of the problem-solving processes that were observed.

The recordings were then analyzed to determine how long each participant spent on each step in the solution of each problem. The percentage of the total solution time for each step was calculated. The relative times spent on the various steps in each problem's solution were then compared, for each participant, in order to see which steps were the most difficult. Underlying this analysis was the assumption that the difficulty posed by each step was proportionate to the time required to deal with it.

Experts often approach problems differently than novices (Chi *et al.*, 1982). Although none of the participants were novices, clearly participants C, E, F, G, and H had less experience in physics than participants A, B and D. In order to determine whether varying degrees of expertise has greater influence on particular aspects of physics problem-solving than on others, I ordered the steps in the various solutions according to their relative difficulty for each participant and then compared the relative difficulty of the different steps in the problem-solving processes of the various participants for each question.

The solutions to the well-defined problems were then compared with those of the ill-defined problem in order to see whether there was a qualitative difference in the ways the problems were solved and the difficulty posed by each step in the problem-solving process.

Results

Steps in the problem-solving process.

Analysis of the recordings showed that all the participants incorporated similar steps in their solutions. While Bransford and Stein's (1984) IDEAL problem-solving model provided a reasonable initial description of these solution processes, it needed to be modified and elaborated in order to improve its description of the particular solution processes that were analyzed. The changes to the IDEAL model are presented in Table 1.

Table 1.
IDEAL Problem-Solving Model and Observed Problem-Solving Processes

IDEAL	Observed Processes
Identify the problem.	Read the problem description (**read**)
Define and represent the problem	Draw representations and given information (**represent**)
Explore possible strategies	Construct solution strategy (**strategize**)
Act on the strategies	Formulate algebraic statements (**equation**)
	Substitute numerical values (**calculate**)
Look back and evaluate the effect of your activities	Evaluate solution (**evaluate**)

According to IDEAL, the first step in solving a problem is identifying the problem, or noticing that a problem exists at all. In this study, the participants were presented with written descriptions of the problems and didn't have to identify the existence of the problems on their own. Therefore, I replaced IDEAL's "identify" step with a "reading problem" step.

Most problems can be represented in multiple ways. Some of these representations can assist in clarifying and emphasizing aspects of the problem that are central to its solution. The types of representations that were used by the participants were: a) writing down the problems' given numerical information, b) drawing free-body-diagrams, and c) sketching problems' geometrical settings. The drawing of these three external representations of the problems was considered the equivalent of IDEAL's "define and represent" stage.

According to IDEAL there is a stage at which the problem-solver "explores" various strategies for solving the problem and chooses one from them. The participants in this study explored various strategies only when they seemed unsure of how to go about solving the problems. In the three well-defined problems the choice of how to solve them was often almost automatic, occurring so quickly that no other options were explicitly considered. Often the participants explained how they were going to solve the problems as they were doing so. The only participants who spent time explicitly exploring various options for solving the well-defined problems were those who did not succeed in solving them. Only in the solution of the ill-defined problem did all the participants explicitly spend time strategizing on how to proceed. I decided to rename IDEAL's "explore" step as "construct solution strategy."

Once a solution strategy had been chosen, many participants developed mathematical equations describing the problem and the model used to solve it (Newton's 2nd Law of motion, the conservation of momentum, etc.). While these equations are representations of the problems and as such their development could be considered part of the "draw representations and given information" step, I chose to list them as a separate step in problem-solving process because they usually occurred at later stages in the solutions. These equations were developed as either general mathematical statements, using variables which were substituted for values later on, or else they were developed as specific numerical statements that incorporated the values taken from the problems' given information from the very start. Although the development of these equations and the use of them to calculate numerical results are the equivalent of IDEAL's "act on strategy" stage, I treated them here as two distinct steps, “formulate algebraic representations” and “substitute numerical values”, since some of the participants clearly made the same distinction. I combined these two steps when the participant made no distinction between them.

IDEAL’s “look back and evaluate solution” was apparent in most of the solutions. Some of the participants were continuously evaluating their solutions while they were constructing them, not only when they had received a final result.

The well-defined problems: Individual problem-solving strategies.

After reading each problem, participant A moved immediately to writing down equations or making a drawing. He did not seem to require any time to strategize out how to solve them. He only explained his solution strategy after being asked, “What are you doing?” He often explained what he was doing as he was doing it. The only time he explicitly evaluated his solution was when there was something that seemed unusual or unexpected. In problem 3 this occurred when considering the units of the 0.15 in the given formula for the kinetic coefficient of friction. Other than in this occurrence, his solutions seemed to require no conscious effort on his part. There were several incidences when participant A combined different stages in the problem-solving process, engaging in both of them simultaneously. For example, the equation he developed in problem 1 was based, from the very start, on the numerical values given in the problem rather than on general variables. Figure 1 is a temporal representation of participant A’s solution to problem number 3.

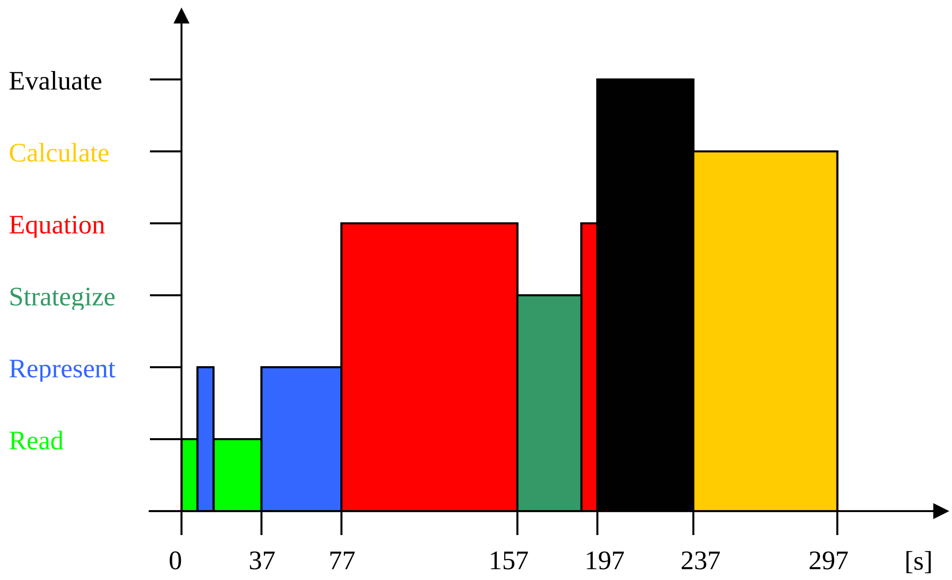


Figure 1: Participant A's Solution to Problem #3

As Figure 1 shows, Participant A's solution to problem #3 proceeded linearly from one step to the next in the sense that there was no explicit looking back; once a step was done it was not returned to. Very little time was spent strategizing and this occurred only after being prompted by me.

Participant B read through the problems quickly and did not return to the written text again. He invested little time in constructing graphic representations of the problems. He did not seem to be solving the problem but rather explaining, as he would to a student, how he would expect the problem to be solved. Thus he often spoke in general terms, elaborated on the reasons why his suggested solution procedures were reasonable, and did not actually calculate a final numerical result for any of the problems, preferring to leave the final result in algebraic form. He did not explicitly evaluate any one of his solutions to the three well-defined problems. He called the first two problems "typical of textbooks" and expected that his students would be able to solve them.

Participant C invested much time in the representation process. She would read a sentence from the text and translate it into another representation. Then she would read another sentence and translate this sentence as well. She explained that she does this on purpose so that she does not have to go back to the written text. Only when this translation process was complete did she begin to construct a solution strategy. Much of her strategizing occurred while creating further representations. Figure 2 is a temporal representation of participant C's solution to problem number 3.

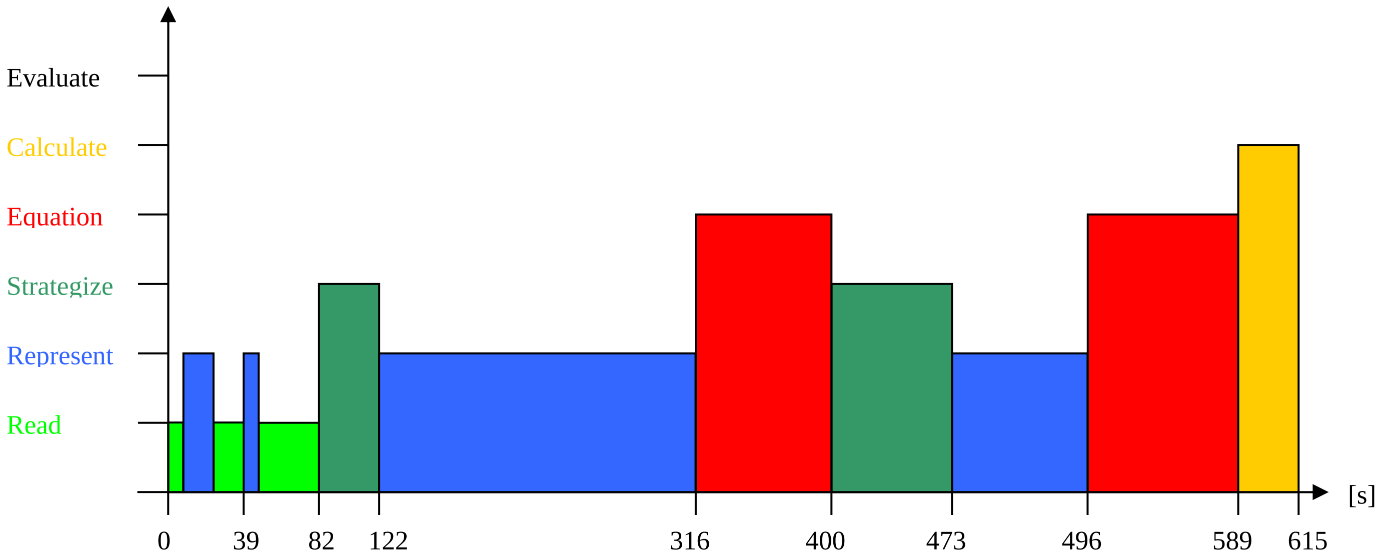


Figure 2: Participant C's Solution to Problem #3

Like participant C, participant D would read a sentence or part of a sentence and then either make a drawing or write down a numerical given, read another section, write down another given, and so on, until he had read through the entire text. He thought that the first question might be difficult for some students because it involved multiple steps and multiple unit transformations. On the other hand, he found the second question straightforward. He did not succeed in solving the third question, even though I gave him a hint midways. He was confused by the speed-dependency of the coefficient of friction. His attempt to solve this problem was characterized by periods of silence that had been absent when he solved the first two problems.

Participant E was the most methodological of all. He worked slowly and carefully, making careful drawings. He spent more time on the problems than any other participant. He too wrote down all the givens and did not return to the written text. He developed general equations as far as possible before replacing variables with numerical values. He was very clear and precise about explaining what he planned on doing. Unlike several other participants, he did not strategize while writing; he explained what he planned on doing in a very clear and precise manner and then proceeded with his plan. He did not combine stages together. He incorporated all six problem-solving stages into all of his solutions.

Participant F, like participant E, described very clearly what he planned on doing. Unlike participant E, he wrote down initial equations but did not develop them. He said he had solved many problems like these as a student and as a teacher.

Participant G did not think ahead to see what would be a useful solution strategy. At each step she considered what might be done at that given stage with the information she had, did it, and then thought about what could be done now, and so on. She did not succeed in solving the second and the third problems. Much of her time was spent trying to recall formulas that might assist her. Her struggle with these two problems was characterized by several periods of silence as she contemplated what to do next. Often she reread the text as if looking for a clue that might give her a direction. Apparently she did not retain a deep enough understanding of Newtonian mechanics in order to deal successfully with these problems.

<u>Well-defined problems: General patterns.</u>

Tables 2, 3, and 4 present the times in seconds that the participants spent engaged in the different stages of problems 1, 2, and 3 (Chi, 1997).

Table 2.

Time Distribution in Seconds for Problem 1

<table>
<tr><th></th><th>Reading</th><th>Representing</th><th>Strategizing</th><th>Developing Equation</th><th>Calculating</th><th>Evaluating Result</th><th>Total Time</th></tr>
<tr><td>Participant A</td><td>33</td><td>-</td><td>20</td><td colspan="2">199</td><td>-</td><td>4:12</td></tr>
<tr><td>Participant B</td><td>20</td><td>-</td><td>33</td><td>77</td><td>-</td><td>-</td><td>2:10</td></tr>
<tr><td>Participant C</td><td>52</td><td>105</td><td>81</td><td>47</td><td>297</td><td>-</td><td>9:42</td></tr>
<tr><td>Participant D</td><td>29</td><td>19</td><td>46</td><td colspan="2">100</td><td>49</td><td>4:03</td></tr>
<tr><td>Participant E</td><td>43</td><td>72</td><td>79</td><td>72</td><td>415</td><td>36</td><td>11:57</td></tr>
<tr><td>Participant F</td><td>37</td><td>24</td><td>54</td><td>13</td><td>128</td><td>81</td><td>5:37</td></tr>
<tr><td>Participant G</td><td>19</td><td>19</td><td>89</td><td>39</td><td>147</td><td>6</td><td>5:19</td></tr>
</table>

Table 3.

Time Distribution in Seconds for Problem 2

<table>
<tr><th></th><th>Reading</th><th>Representing</th><th>Strategizing</th><th>Developing Equation</th><th>Calculating</th><th>Evaluating Result</th><th>Total Time</th></tr>
<tr><td>Participant A</td><td>33</td><td>30</td><td>18</td><td>17</td><td>59</td><td>-</td><td>2:39</td></tr>
<tr><td>Participant B</td><td>24</td><td>8</td><td colspan="2">52</td><td>90</td><td>-</td><td>2:54</td></tr>
<tr><td>Participant C</td><td>33</td><td>51</td><td>28</td><td>30</td><td>112</td><td>-</td><td>4:14</td></tr>
<tr><td>Participant D</td><td>27</td><td>9</td><td>27</td><td>19</td><td>76</td><td>-</td><td>2:38</td></tr>
<tr><td>Participant E</td><td>27</td><td>52</td><td>38</td><td>66</td><td>79</td><td>11</td><td>4:33</td></tr>
<tr><td>Participant F</td><td>17</td><td>24</td><td>26</td><td>7</td><td>69</td><td>-</td><td>2:23</td></tr>
<tr><td>Participant G</td><td>45</td><td>-</td><td>181 (Including silences)</td><td>45</td><td>77</td><td>-</td><td>5:48</td></tr>
</table>

Table 4.

Time Distribution in Seconds for Problem 3

	Reading	Representing	Strategizing	Developing Equation	Calculating	Evaluating Result	Total Time
Participant A	30	48	32	89	59	39	4:57
Participant B	24	82	24	106	-	-	3:56
Participant C	62	202	111	191	49	-	10:15
Participant D	34	26	257 (Including silences)	5	107	-	7:09
Participant E	60	114	102	270	24	30	10:00
Participant F	25	30	74	22	96	-	4:07
Participant G	17	22	93 (Including silences)	54	-	-	3:06

When we look at the results of temporal analyses of the three well-defined problems, a number of patterns become apparent:

1. In general, “reading” was the shortest stage, taking on average 10%, 14%, and 10% of the solution time in the first, second, and third problems respectively.
2. In problems 1 and 2, where there was little place for equation manipulation, the participants spent most of there time calculating a numerical result. In problem 3 many participants shifted their emphasis to equation manipulation.
3. More participants evaluated their answers to problem 1 than to the other problems. This is because this problem, unlike the other two, involved the conversion of units, of which several participants were unsure.
4. Participants A and B, the professors, spent significantly less time strategizing in problems 1 and 3 than the other participants. Problem 2 was the most familiar of the three problems. All the participants said that they had solved many, many problems that were very similar to problem 2. While this was true of problem 1 as

well, the geometry and units involved made the problem a bit less recognizable. The block sliding down the inclined plane in problem 3was also a very familiar setting, but not for terminal velocity problems.

The ill-defined problem: Individual problem-solving strategies.

Participant A was the only one who fully solved the ill-defined problem. While solving the well-defined problems he was quick and self-confident. Now, he was pensive, at times silent. Often he seemed to be speaking to himself rather than to me. For example: "I know the density of water but have no idea how much rain is hitting my umbrella per second and how fast it is coming down…ta ta ta…raindrops are falling on my head…interesting question…" He realized very quickly that he needed to constrain the problem: "I'm going to make a number of assumptions. Rain comes down in streaks… I'm going to do an approximation of continuous flow…I need to think of the velocity of a cylinder of water coming down on the umbrella…How high are the clouds? Free fall… I'm going to throw in a number – 200m. But that's wrong, irrelevant, because [the raindrops] quickly reach terminal velocity. At what speed does rain fall on my head? Interesting…hmmm…like parachutes."

Rather than planning out in advance what was needed in order to solve the problem, he broke the problem down into sub-problems and dealt with each independently, conducting an interim summary at the end of each sub-problem: "What do I have so far? This is the total mass hitting per second…If I multiply this by v [the speed of the raindrops] I have the force on the umbrella." The interim summaries included a check of the general behavior of the equations he had developed, such as" "What would happen if a [the radius of a raindrop] became very small…?"

Drawings were made after periods of silence or of developing a strategy rather than after the reading of the text. In the interview that followed the problem-solving session, participant A said that the drawings were not an aid to guide his thinking, but rather that he drew them only after he had decided how to proceed and then he used them to make sure he wasn't confusing directions, vector components, geometries, etc. This would imply that he had decided how to solve the well-defined problems already while reading the text, since he made his drawings immediately after he finished reading. Figure 3 is a temporal representation of participant A's solution to the ill-defined problem.

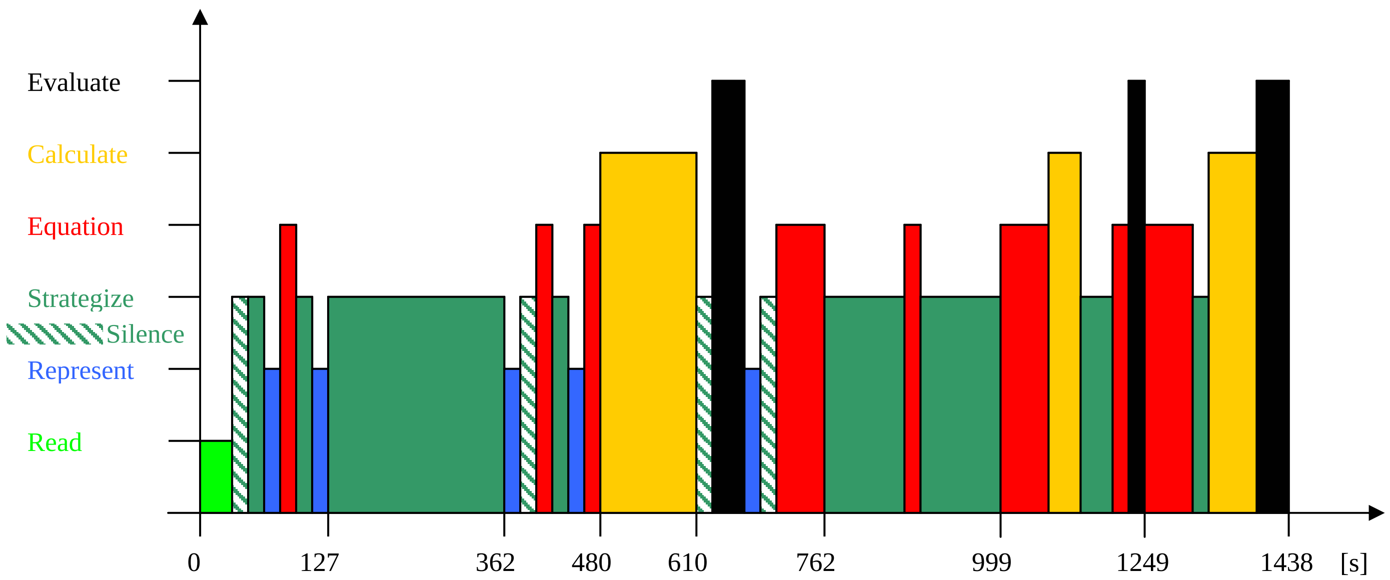


Figure 3: Participant A's Solution to the Ill-Defined Problem

As Figure 3 demonstrates and unlike his solutions to the well-defined problems, participants A's solution to the ill-defined problem did not proceed linearly from one step to the next. He divided the problem into 3 sections and worked on each section separately, but within each section he jumped back and forth between strategizing and developing equations. In stark contrast with the well-defined problems, much of his time here was spent strategizing. Each section ended with an explicit evaluation stage.

Participant B began by explaining that he thought the problem of estimating the force of the rain striking the umbrella was irrelevant. He said that based on personal experience the typical failure mode of an umbrella was due to wind force and not rain force. If at all, he expected that the distribution of the rain force across the surface of the umbrella rather than its total downward force would be important. Thus he recommended expanding the scope of the problem. He then proceeded to consider which quantities determine the total downward force: “Let’s say we have rain falling at a certain rate, where a certain amount of mass strikes the umbrella…a certain amount of mass falls per second per unit area…all the forces in this problem will scale with this…they’ll also scale with the velocity of the rain, because this is a momentum transfer problem like we had in the earlier collision problem.” He then discussed the value of assumptions: “Lot’s of physicists would first calculate a kind of a back-of-the-envelope type calculation where they would take a circular umbrella …you might assume then for the velocity of the rain some number which is characteristic of the kind of terminal velocity that raindrops might reach…now there isn’t really a number that is characteristic of the terminal velocity of raindrops, instead there’s a range of numbers, I suppose…in would range all the way from fog where the drops are suspended in the air to heavy rain where the drops fall at, let’s say, 30 – 40 meters per second…so you could assume a value for the rain’s velocity, but there isn’t any real point in doing that. What we should do instead is just calculate an expression that will give an understanding of the scaling, how does the force depend on the nature of the rain.” However, he did not proceed to develop such an expression but instead returned to the importance of considering the bending moments created by the rain’s distribution over the umbrella. Again, like in the well-defined problems, there was a sense that he was explaining how the problem should be solved rather than actually solving it.

Participant C was one of the few who noticed a similarity between the ill-defined and one of the well-defined questions. When considering how to calculate the force a single raindrop exerts on the umbrella, she said: “Raindrops don’t bounce…It’s like a little impact thing, like the problem we just did, with the ball bouncing…” She realized she needed data beyond what was given by the problem but didn’t know how to generate it. She thought of doing experiments as a way of generating the missing data. The possibility of making subjective assumptions did not occur to her: “How do you figure out the mass of a typical raindrop? I could go out in the rain [laughing to herself] and catch one and measure its volume! Or I could take an eye-dropper and drop a typical drop of water… I have no sense of typical sizes; I’m not good at these sorts of things.” Her attempted solution was characterized by many periods of silence, going back and forth between strategizing and drawing. After a bit over 14 minutes she gave up, saying: “I don’t like this question because there are too many things I don’t know.” Figure 4 is a temporal representation of her solution to the ill-defined problem.

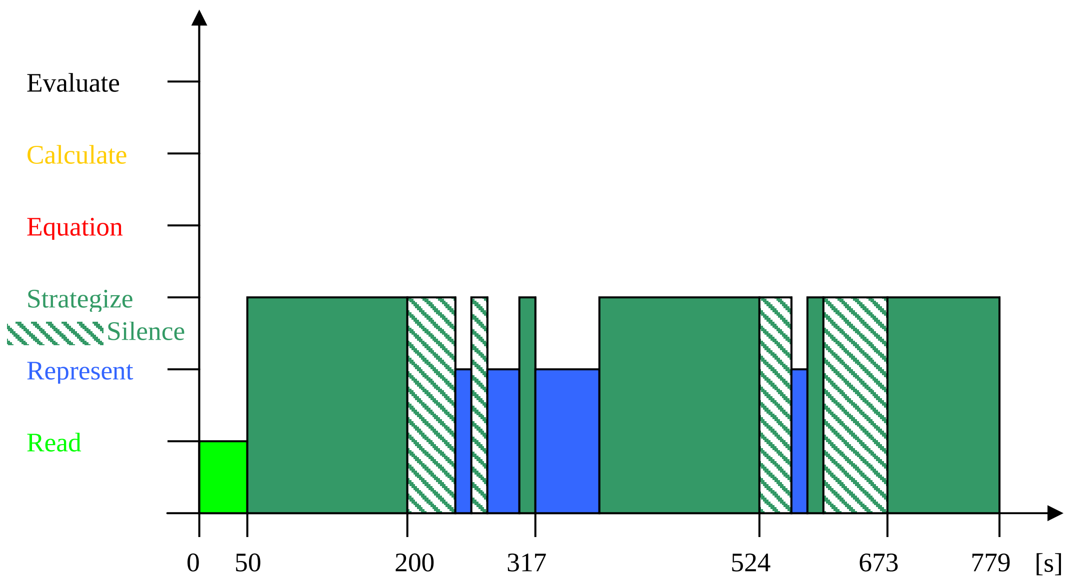


Figure 3: Participant A’s Solution to the Ill-Defined Problem

Participant D also thought that the force of the rain was probably unimportant: “My hunch would be that if we would figure that out [the force of the rain on the umbrella] it would be very small, and that the force on an umbrella does not really depend on the rain.” He realized he needed data of the speed of the rain and its impact time, but did not consider the possibility of making assumptions that would give him rough estimates of these values. Instead he thought of ways to find validated values: “I need to look up or ask someone or measure the speed of a raindrop falling…I would need to use a camera to determine how long it takes a raindrop to stop [when hitting an umbrella]. Or I could drop drops of water on a plate and use a force probe to determine the force applied by a single raindrop.” Like participant C, participant D too was drawn to the option of doing experiments to obtain the needed data. His strategizing ignored the importance of the amount of water falling. He made no progress in solving the problem beyond sketching an umbrella and discussing ways to find missing data.

Participant E begin by critiquing the narrowness of the problem: “Based on my personal experience, the failure mode of an umbrella is not caused by the rain falling but by the wind blowing.” He then discussed the need for additional information: “Since I don’t really know the umbrella’s size, what I really need to figure out is the pressure applied to the umbrella…to do this I need to know how to characterize rain flux. I need to know the speed of falling rain and what is known about the amount of rain that comes down in different kinds of storms. I’m not sure how to figure that out…One way to do this would be to come up with some guesses or to look up info on rainfall.” Although participant E knew what he needed to know and realized that making guesses could be a reasonable way of obtaining this information, he did not make any assumptions or estimates.

Participant F knew what needed to be done but couldn’t do it. For example, he said: “I don’t think I can solve this problem under these circumstances without knowing the size and shape of the umbrella. So I'll need to make some assumptions about these." However, he never made these assumptions. In another case: "I would take the biggest possible rainfall and largest possible umbrella… I need to go with the limiting case. I'll imagine I'm standing with the umbrella under a vertical river. I need to calculate the total impulse. Force is impulse per time. I need to make assumptions about how fast [the] water is falling and the shape of the umbrella." Again, while he realized the need to make assumptions, he never actually made them. His attempted solution of this problem, unlike his solutions to the three well-defined problems, was characterized by several periods of silence, of re-reading the text.

Participant G, after reading the question, said: "This is a question I hope my engineering friends could answer. It's not a physics problem but rather a design question; it's wide open…Only a cruel professor would give this to his physics students…I've not seen questions of this sort. I think this would have been more interesting than the questions I saw as a student."

<u>The ill-defined problem: General patterns.</u>

The most prominent feature that is immediately apparent is that only participant A succeeded in solving the problem! While participants B through F strategized how they would go about solving the problem and generated models that could have perhaps led to a successful solution, they never put their strategy to test; they never actually developed a mathematical representation of the problem. Participant B wrote down a general relation that described the relation between the mass flux and pressure, but never elaborated and developed this relation so that it would correspond with the features of the problem. Participant G never got beyond the stage of reading, since, as the analysis of the well-defined problems showed, she lacked the necessary content knowledge.

Another major feature was the existence of periods of silence, with the participants contemplating how to proceed. Although the participants were supposed to "think-aloud", at times the cognitive demand of the problem facing them was too great for them to think and talk about it simultaneously. When questioned about these periods of silence, all the participants responded that they had felt the need to "straighten out their thoughts." These periods of silence occurred in the solutions to the well-defined problems only when the participants did not know how to proceed did. Clearly, solving the ill-defined problem was much more difficult than solving the well-defined problems.

An additional interesting finding is that the process of solving this problem was nonlinear. When solving the well-defined problems, the participants proceeded linearly from one step to the next, seldom returning to an earlier step to modify, add, or amend something they had done then. On the other hand, participant A jumped back and forth from between "strategizing" and "developing an equation" while dealing with the ill-defined problem. Participant D jumped back and forth between the "strategizing" stage and the "reading" stage, looking for additional information, just as participant G had done while attempting to solve the second problem. Clearly he felt he was missing something necessary to solve the problem.

Table 5 presents the time the participants spent engaged in the different stages of problem 4, the ill-defined problem.

Table 5.

Time Distribution in Seconds for the Ill-Defined Problem

	Reading	Representing	Silence	Strategizing	Developing Equation	Calculating	Evaluating Result	Total Time
Participant A	39	4 + 2 + 3 + 10 + 9 = 28	30 + 13 + 12 + 19 + 28 = 102	27 + 19 + 235 + 16 + 117 + 107 + 55 + 27 = 603	6 + 5 + 33 + 61 + 13 + 73 + 35 + 63 = 289	130 + 55 + 60 = 245	51 + 32 + 49 = 132	23:58
Participant B	34	13 + 9 = 22	22 + 11 + 9+ 20 + 12 = 74	63+ 51 + 15 + 22 + 12 + 42 + 105 + 46 = 356	36	-	-	8:42
Participant C	50	13 + 25 + 63 + 7 = 108	36 + 44 + 60 = 140	150 + 35 + 144 + 7 + 106 = 442	-	-	-	12:20
Participant D	40	46	50 + 21 + 21 = 92	42 + 46 + 94 = 182	-	-	-	6:00
Participant E	46 + 9 = 55	66 + 30 = 96	9 + 20 + 25 = 54	80 + 50 + 120 + 360 = 610	-	-	-	13:35
Participant F	46 + 9 = 54	66 + 30 = 96	9 + 20 + 25 = 54	80 + 50 + 120 = 360 = 610	-	-	-	13:34
Participant G	-	-	-	-	-	-	-	-

When we look at the results of temporal analysis of the ill-defined problem, a number of additional patterns become apparent:

1. The relative time spent on strategizing on the ill-defined problem was much greater than on the well-defined problems. The ill-defined problem was more difficult to solve than the well-defined problems, and because of its greater cognitive demand the participants needed to spend a larger portion of their time trying to make sense of the problem and constructing a solution strategy.
2. Only participant A really went beyond strategizing and acted on his strategy. All the other participants spoke of what they would do, what they needed to solve the problem, but none actually did anything other than strategize. Participant B wrote down a very general equation but did not develop it so that it would describe the specific conditions of this problem.
3. Little time was spent drawing representations.

Discussion

Shin, Jonassen, and McGee (2003) showed that while domain-specific knowledge is needed to solve both well- and ill-defined problems in astronomy, ill-defined problems in astronomy apparently require an additional type of reasoning that students do not have because of the apparent limitations of their educational experiences. They did not investigate the characteristics of this type of reasoning. Assuming that there is no fundamental difference between the cognitive processes involved in solving problems astronomy or Newtonian mechanics, the present study demonstrates that the cognitive skill that is needed to solve ill-defined problems and is not developed in conventional educational settings is the ability to conceive of and apply subjective assumptions that constrain the solution space of an ill-defined problem and convert it into a well-defined problem. Most of the participants in the present study had the domain-specific knowledge needed to solve the ill-defined problem; many of them realized the need to make assumptions. However, only the two participants who had prior experience working with ill-defined problems made any relevant assumptions and only one of them implemented these assumptions.

The implication of this finding is that if we want students to be able to use their understanding of physics in making sense of and solving real-world problems we need to either give them experience applying their knowledge in real-world contexts or to develop learning environments in which students can construct both domain-specific knowledge and skill at analyzing and constraining ill-defined problems. Examples of such learning environments exist: Design-Based Science (DBS) (Fortus *et al.*, 2004) and Learning By Design (LBD™) (Kolodner *et al.*, 2003) curricula are structured around design problems, while Project-Based Science (PBS) (Krajcik *et al.*, 2003) units are based on complex, realistic driving questions. Indeed, students who constructed an understanding of a variety of scientific concepts while participating in DBS enactments, not just Newtonian mechanics but also electrochemistry, thermodynamics and wave mechanics, were able to transfer their knowledge to new design problems that were based on similar physical concepts (Fortus *et al.*, 2002). Since the vast majority of traditional textbooks are based on well-defined problems, this finding supports and underscores the need for a new generation of curriculum materials that are based on complex real-world scenarios.

Another interesting finding is that few of the participants noticed any connection between the ill-defined problem and the well-defined problems that they had just solved. This is reminiscent of other studies (Gick & Holyoak, 1980; Reed *et al.*, 1974) in which participants were not able to apply the knowledge they had just constructed to a new, analogous situation. But unlike those studies where it was questionable whether the participants had constructed an abstract and generalized, and therefore transferable understanding of the needed concepts, or whether they just hadn't noticed the underlying similarity between the source and transfer problems, clearly here what was missing was recognition of the underlying similarity.

While solving the well-defined problems, none of the participants strayed beyond the context defined by these problems. On the other hand, several of the participants elicited images and connections to other contexts while working on the ill-defined problem. For example, participant A started humming the song "Raindrops are falling on my head." Participant F imagined himself standing with the umbrella under a waterfall. Several participants discussed the relative importance of considering the wind. While only participant A succeeded in solving the ill-defined problem, all the participants except participant G stated in the interviews that followed the problem-solving sessions that they found the ill-defined problem more interesting that the well-defined ones. They liked its open-endedness, it realism and lack of academic sterility. They wished that they had been given more problems of this sort during their physics education.

Is it possible that this interest was based in part on the connections they had made between the problem and other experiences they had, imaginary or real? In any case, since one of the prime goals of education is to help students construct knowledge and skills that are useful and useable after school is over (Broudy, 1977), we need to help students deeply integrate new ideas into existing cognitive networks so that there will be many possible mental paths that will involve these new ideas; using and learning new concepts in the context of ill-defined problems seems to encourage the construction of multiple connections to the new concepts.

While solving the well-defined problems, the participants required little strategizing time. The path between comprehending the goal of these problems and proceeding to solve them was very short. This almost automatic application of domain-specific knowledge follows the characteristics of low-road transfer as laid out by Salomon and Perkins (1989): "… spontaneous, automatic transfer of highly practiced skills, with little need for reflective thinking." On the other hand, the solutions to the ill-defined problem were characteristic of high-road transfer: "… the explicit conscious formulation of abstraction in one situation that allows making a connection to another" (p. 118). The reason different types of transfer were involved in these problems has to do with the amount of experience the participants had in solving well-and ill-defined problems. In low-road transfer, a skill is learned and practiced in a variety of contexts until it becomes automatic and flexible because of the variety of the practice contexts. On a later occasion, the characteristics of a new context sufficiently resemble those of one of the earlier contexts in which the skill was learned and practiced, and the skill is automatically activated. The fundamental characteristics of low-road transfer are therefore *varied practice* and *automaticity*. Thus, we should not expect low-road transfer of an idea or skill until it has been applied many times in a wide variety of situations. Until then it can only be transferred by the high-road. This is the reason reading is so difficult for new readers – the cognitive demand of applying the skill of reading is so great that they have little cognitive reserve left to process the content of what they are reading. Only after they have spent hundreds of hours reading does the process of reading start to become automatized, leaving greater cognitive power available to comprehend the content being read.

This implies that the more exposure to and experience with ill-defined problems that we give to students, the easier it should become for them to cope with these problems. Again, the conclusion is that we need to incorporate many more ill-defined problems into our curricula.

References

AAAS. (1990). *Science for all americans*. New York: Oxford University Press.

Bransford, J. D., & Stein, B. S. (1984). *The ideal problem solver: A guider for improving thinking, learning, and creativity.* New York: Freeman.

Broudy, H. S. (1977). Types of knowledge and purposes of education. In R. C. Anderson, R. J. Spiro & W. E. Montague (Eds.), *Schooling and the acquisition of knowledge* (pp. 1-17). Hillsdale, NJ: Erlbaum.

Chi, M. T. H. (1997). Quantifying qualitative analyses of verbal data: A practical guide. *The Journal of the Learning Sciences, 6*(3), 271-315.

Chi, M. T. H., Glaser, R., & Rees, E. (1982). Expertise in problem solving. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Ericsson, K. A., & Simon, H. A. (1984). *Protocol analysis: Verbal reports as data*. Cambridge, MA: MIT Press.

Evans, J. S. B. T. (1989). *Bias in human reasoning*. Hillsdale, NJ: Erlbaum.

Fortus, D., Dershimer, R. C., Krajcik, J. S., & Marx, R. W. (2002). *Design-based science and real-world problem-solving.* Paper presented at the The 2002 Annual Meeting of the American Educational Research Association, New Orleans.

Fortus, D., Dershimer, R. C., Krajcik, J. S., Marx, R. W., & Mamlok-Naaman, R. (2004). Design-based science (dbs) and student learning. *Journal of Research in Science Teaching, 41*(10), 1081-1110.

Frederiksen, N. (1986). Toward a broader conception of human intelligence. In R. J. Sternberg & R. K. Wagner (Eds.), *Practical intelligence: Nature and origins of competence in the everyday world* (pp. 84-116). New York: Cambridge University Press.

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive Psychology, 12*(3), 306-355.

Glass, A. L., & Holyoak, K. J. (1986). *Cognition*. New York: Random House.

Glass, A. L., Holyoak, K. J., & Santa, J. L. (1979). *Cognition*. Reading, MA: Addison-Wesley.

International Association for the Evaluation of Educational Achievement., & TIMSS International Study Center. (1996). *Science achievement in the middle school years: Iea's third international mathematics and science study (timss)*. Chestnut Hill, MA: Center for the Study of Testing Evaluation and Educational Policy Boston College.

Kolodner, J. L., Camp, P. J., Crismond, D., Fasse, B. B., Gray, J., Holbrook, J. K., et al. (2003). Problem-based learning meets case-based reasoning in the middle-school science classroom: Putting learning by design™ into practice. *The Journal of the Learning Sciences, 14*(4), 495-547.

Krajcik, J. S., Czerniak, C., & Berger, C. (2003). *Teaching children science in elementary and middle school classrooms: A project-based approach*. New York: McGraw-Hill College.

Lapointe, A., Mead, N., & Phillips, G. (1989). *A world of differences*. Princeton, NJ: Educational Testing Services.

Newell, A. (1969). Heuristic programming: Ill-structured problems. In J. S. Aronofsky (Ed.), *Progress in operations research* (Vol. 3). New York: John Wiley and Sons, Inc.

Newell, A., & Simon, H. A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall.

Nickerson, R. S. (1994). The teaching of thinking and problem solving. In R. J. Sternberg (Ed.), *Thinking and problem solving* (pp. 409-449). San Diego, CA: Academic Press.

Reed, S. K., Ernst, G. W., & Banerji, R. (1974). The role of analogy in transfer between similar problem states. *Cognitive Psychology, 6*, 436-450.

Reitman, W. (1964). Heuristic decision procedures, open constraints, and the structure of ill-defined problems. In M. W. Shelley & G. L. Bryan (Eds.), *Human judgments and optimality* (pp. 282-315). New York: John Wiley and Sons, Inc.

Roberts, P. (1995). The place of design in technology education. In D. Layton (Ed.), *Innovations in science and technology education, vol. V* (pp. 171-179): UNESCO Publishing.

Salomon, G., & Perkins, D. (1989). Rocky roads to transfer: Rethinking mechanisms of a neglected phenomenon. *Educational Psychologist, 24*(2), 113-142.

Shin, N., Jonassen, D. H., & McGee, S. (2003). Predictors of well-structured and ill-structured problem solving in an astronomy simulation. *Journal of Research in Science Teaching, 40*(1), 6-33.

Simon, H. A. (1973). The structure of ill-structured problems. *Artificial Intelligence, 4*, 181-201.

Simon, H. A. (1978). Information-processing theory of human problem solving. In W. K. Estes (Ed.), *Handbook of learning and cognitive processes* (Vol. 5, pp. 271-295). Hillsdale, NJ: Erlbaum.

Appendix

Problem 1.

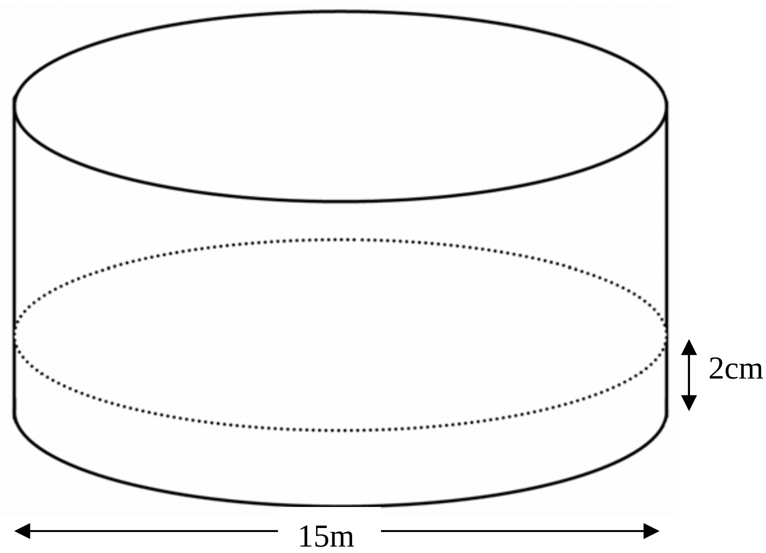
ΔV – Volume of water added to pool

d – pool diameter = 15m

h – height of water added to pool = 2cm

v – volume of pail = 10 liter

$\dot{n}$ – number of pails poured into pool every minute



$$\Delta V = \pi(\frac{d}{2})^2 h = \pi(\frac{15}{2})^2 \cdot \frac{2}{100} = 3.53 m^3$$

$$\dot{n} = \frac{\Delta V}{v} \cdot \frac{1}{60} = \frac{3.53}{10/10^3} \cdot \frac{1}{60} = 5.9 \frac{pails}{\min}$$

Problem 2.

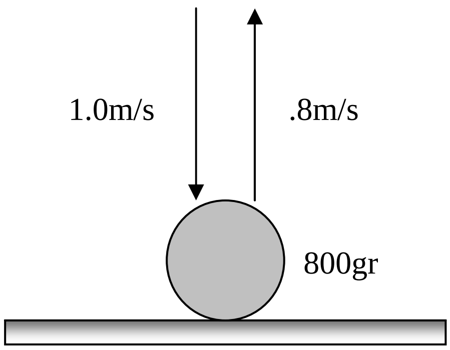
m - the ball's mass = 800gr

v - the ball's impact speed = 1.0m/s

u - the ball's recoil speed = .8m/s

Δt - the impact time = .05s

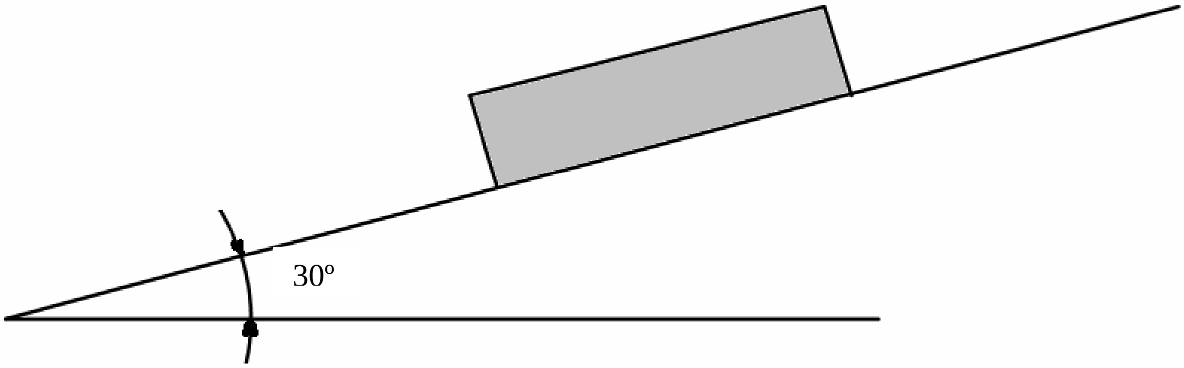
$\vec{F}$ - the average net force applied to the ball during the impact



$$\overline{F} = \frac{|\Delta(m\vec{v})|}{\Delta t} = \frac{m(v+u)}{\Delta t} = \frac{800/1000 \cdot (1.0+0.8)}{.05} = 28.8N$$

Problem 3.

A block is pushed in order to start it sliding down an inclined plane with an angle of 30˚. The coefficient of friction between the plane and the block is velocity dependent: it is equal to $\mu_k = 0.15v$, where v is the block’s speed in m/s relative to the plane. Assuming the plane is very long, what is the maximum speed relative to the plane that the block can attain?



α - the slope of the inclined plane = 30º

μ_k - the kinetic friction coefficient

N - the normal force between the block and the inclined plane

a – the blocks downward acceleration parallel to the inclined plane

f_k - the kinetic friction between the block and the inclined plane

g - the gravitational acceleration = 9.8m/s^2

$$N = mg\cos\alpha$$

$$ma = mg\sin\alpha - f_k = mg\sin\alpha - \mu_k N = mg\sin\alpha - \mu_k mg\cos\alpha = mg(\sin\alpha - \mu_k\cos\alpha)$$

$a = g(\sin\alpha - \mu_k\cos\alpha) = 0$ when terminal speed is reached

$$\mu_k = 0.15v = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

$$v = \frac{\tan\alpha}{0.15} = \frac{\tan 30^0}{0.15} = 3.85m/s$$

Problem 4.

1. Assume all raindrops are perfect spheres with identical diameters – d.
2. Assume the umbrella is a circle of diameter D.
3. Assume the rainfall is such that H meters of rain fall every 24 hours.
4. Assume that the raindrops have reached their terminal velocity v_{impact} before striking the umbrella.
5. Assume that the raindrops' collision with the umbrella is completely inelastic, that is, the recoil velocity u is null – u = 0m/s

$\overline{F}$ - the average net downward force applied to the umbrella because of the rain falling on it

ρ_{water} - the density of water

$\dot{m}$ - the mass of the rain falling on the umbrella in 1s

The average force of the rain on the umbrella is calculated from impulse-momentum considerations.

$$\overline{F} = \frac{|\Delta\vec{p}|}{\Delta t} = \dot{m}(v_{impact} - u) = \dot{m}v_{impact}$$

$\dot{V}_{umbrella}$ - the volume of water falling on the umbrella in 1s

H - daily rainfall

D - the diameter of the umbrella

$$\overline{F} = \dot{m}v_{impact} = \rho_{water}\dot{V}_{umbrella}v_{impact} = \rho_{water}\left[\pi\left(\frac{D}{2}\right)^2 \frac{H}{24\cdot 3600}\right]v_{impact}$$

6. Assume that the impact velocity is the raindrops' terminal velocity; therefore, the raindrops are in equilibrium before striking the umbrella.

W_{drop} - the weight of a raindrop

F_{drag} - the aerodynamic drag on a raindrop

m_{drop} - the mass of a rain drop

g - acceleration due to gravity

d - the diameter of a raindrop

C_D - the drag coefficient of a raindrop

ρ_{air} - the density of air

$$W_{drop} = F_{drag}$$

$$m_{drop} g = \rho_{water} \tfrac{\pi}{6} d^3 g = \tfrac{1}{2} \rho_{air} v_{impacxt}^2 C_D \left(\tfrac{\pi}{4} d^2\right)$$

$$v_{impact} = \sqrt{\frac{4\rho_{water} dg}{3\rho_{air} C_D}}$$

Raindrops with larger diameters have greater impact velocities. The larger the impact velocity, the greater the impact force. A worse case scenario will have very large raindrops.

7. Assume that the raindrops' diameter is $d = 5 \times 10^{-3} m$.
8. Assume the raindrops' drag coefficient is $C_D = 1$.

The density of water is $\rho_{water} = 1000 \, {}^{kg}\!/_{m^3}$.
The density of air is $\rho_{air} = 1.2 \, {}^{kg}\!/_{m^3}$.
Acceleration due to gravity is 10m/s^2.

$$v_{impact} = \sqrt{\frac{4 \cdot 1000 \cdot 5 \times 10^{-3} \cdot 10}{3 \cdot 1.2 \cdot 1}} \approx 7 m/s$$

9. Assume a daily rainfall equal to the record daily rainfall for California, which is $H = 26in = 0.66m$ (http://www.losangelesalmanac.com/topics/Weather/we15.htm).
10. Assume that an average umbrella's diameter is $D = 1.5m$.

.

$$\overline{F} = \rho_{water} \left[\pi \left(\frac{D}{2} \right)^2 \frac{H}{24 \cdot 3600} \right] v_{impact} = 1000 \left[\pi \left(\frac{1.5}{2} \right)^2 \frac{0.66}{24 \cdot 3600} \right] \cdot 7 = 0.09N$$

This estimated force (0.09N) is negligible.