HABITS OF MIND

COMPUTATION AND ESTIMATION (12B)

In everyday life, people often have to make simple calculations in their minds. The advent of the small, inexpensive electronic calculator has made it possible to carry out a range of calculations with precision and to check the reasonableness of the answers. There are also many situations where an approximate answer is as useful as a precise one. Estimating approximate answers can often take the place of making a precise measurement or a careful calculation or be used to check calculations made using electronic calculators or paper and pencil. Skill in estimation is based on a sense of what level of precision is needed, which in turn depends on understanding the purpose of the calculation.

The map is organized around two strands—*estimation* and *computation*. In elementary grades, the focus is on using numbers to represent quantities and basic operations on numbers. In middle school, the kinds of numbers involved in problems expands to include fractions, decimals, and percents and the kinds of computations expand to include calculation of central tendency and probability. Computer spreadsheets come into play as a way to perform arithmetic operations on numbers. In high school, problem solving includes use of ratio and proportion, simple algebraic formulas, and computer-based analysis. Throughout, students are expected to make sound choices about which tools to use—their heads, paper and pencil, calculators, or computers—and to judge the reasonableness of measurements and calculations.

This map draws on mathematical ideas that also appear on the **MATHEMATICAL PROCESSES** map in *Atlas 1* and the **NUMBERS** map in this volume.

NOTES

The grades 3-5 benchmark "Make calculations when necessary to solve real-world problems…" is central to the overall topic of the map, as indicated by the number of arrows connecting it to other benchmarks. The link between this benchmark and the benchmark "State the purpose of each step in a calculation" indicates that it is not sufficient for students to be able to solve problems simply by memorizing steps in a solution. They are also expected to understand the purpose of each step in all of the calculations they make. Three new benchmarks derived from *Science for All Americans* identify specific skills needed for solving a wide range of problems.

RESEARCH IN *BENCHMARKS*

Research suggests using word problems as a basis for teaching addition and subtraction concepts, rather than teaching computational skills first and then applying them to solve problems (Carpenter & Moser, 1983). Research has identified a developmental progression of concepts and skills that students use for addition and subtraction (Fuson, 1988; Fuson, 1992). There is evidence that instruction based on this progression can help (Romberg & Carpenter, 1986; Fuson & Willis, 1989).

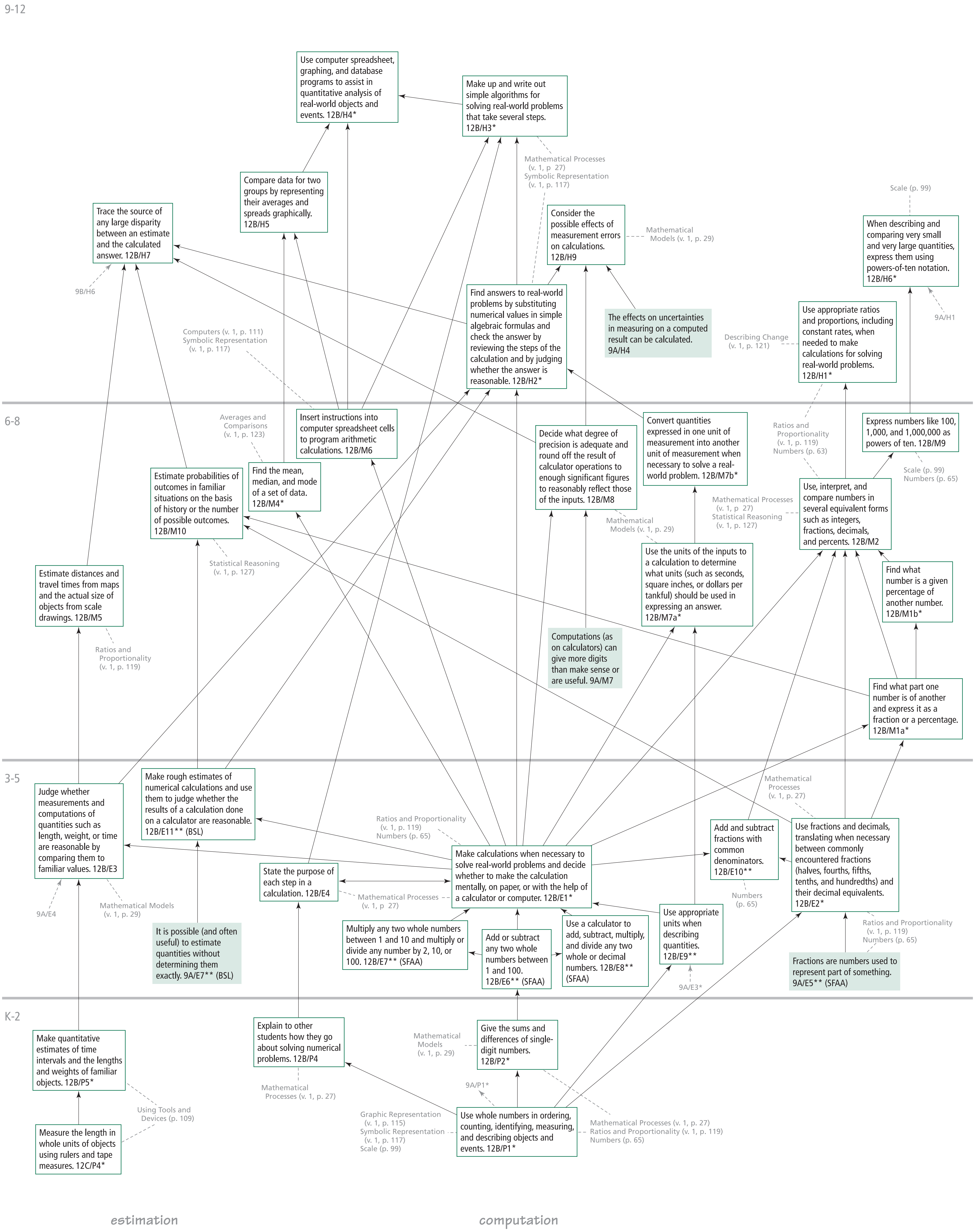
Students make a variety of errors in multi-digit addition and subtraction calculations (Brown & Van Lehn, 1982). Student errors suggest students interpret and treat multi-digit numbers as single-digit numbers placed adjacent to each other, rather than using place-value meanings for the digits in different positions (Fuson, 1992). Research also suggests students interpret multiplication of whole numbers mainly as repeated addition. This interpretation is inadequate for many multiplication problems and can lead to restrictive intuitive notions such as "multiplication always makes larger"(Greer, 1992).

Elementary- and middle-school students make several errors when they operate on decimals and fractions (Benander & Clement, 1985; Kouba et al., 1988; Peck & Jencks, 1981; Wearne & Hiebert, 1988). For example, many middle-school students cannot add 4 + 0.3 correctly or $7\frac{1}{8} + 3\frac{1}{2}$ (Kouba et al., 1988; Wearne & Hiebert, 1988). Also, lower middle-school students may have difficulties understanding the relationship between factions and decimals (Markovits & Sowder, 1991). These errors are due in part to the fact that students lack essential concepts about decimals and fractions and have memorized procedures that they apply incorrectly. Interventions to improve concept knowledge can lead to increased ability by 5^{th}-graders to add and subtract decimals correctly (Wearne & Hiebert, 1988).

Students of all ages misunderstand multiplication and division (Bell et al., 1984; Graeber & Tirosh, 1988; Greer, 1992). Commonly held misconceptions include "multiplication always makes larger" and "division always makes smaller." Students may correctly select multiplication as the operation needed to calculate the cost of gasoline when the amount and unit cost are integers, then select division for the same problem when the amount and unit cost are decimal numbers (Bell et al., 1981). Suggestions have been made to improve student concepts of multiplication (Greer, 1992), but further research is needed to evaluate these suggestions.

The use of calculators in K-12 mathematics does not hinder the development of basic computation skills and frequently improves concept development and paper-and-pencil skills, both in basic operations and in problem solving (Hembree & Dessart, 1986; Kaput, 1992). The use of calculators in testing produces higher scores than paper-and-pencil efforts in problem solving as well as in basic operations (Hembree & Dessart, 1986).

Good estimators use a variety of estimating tactics and switch easily between them. They understand place value and the meaning of operations, and are skilled in mental computation. Poor estimators rely on algorithms that are more likely to yield the exact answer. They lack an understanding of the notion and value of estimation and often describe it as "guessing" (Sowder, 1992b). Before 6th grade, students develop few estimation skills from their natural experiences (Case & Sowder, 1990; Sowder, 1992b). Some researchers caution that teaching estimation to young children may have as its single effect that they master specific procedures in a superficial manner (Sowder, 1992b).


9-12
Use computer spreadsheet, graphing, and database programs to assist in quantitative analysis of real-world objects and events. 12B/H4*
Make up and write out simple algorithms for solving real-world problems that take several steps. 12B/H3*
Mathematical Processes (v. 1, p 27)
Symbolic Representation (v. 1, p. 117)
Scale (p. 99)
Compare data for two groups by representing their averages and spreads graphically. 12B/H5
Trace the source of any large disparity between an estimate and the calculated answer. 12B/H7
Consider the possible effects of measurement errors on calculations. 12B/H9
Mathematical Models (v. 1, p. 29)
When describing and comparing very small and very large quantities, express them using powers-of-ten notation. 12B/H6*
9B/H6
Find answers to real-world problems by substituting numerical values in simple algebraic formulas and check the answer by reviewing the steps of the calculation and by judging whether the answer is reasonable. 12B/H2*
The effects on uncertainties in measuring on a computed result can be calculated. 9A/H4
9A/H1
Computers (v. 1, p. 111)
Symbolic Representation (v. 1, p. 117)
Describing Change (v. 1, p. 121)
Use appropriate ratios and proportions, including constant rates, when needed to make calculations for solving real-world problems. 12B/H1*
6-8
Averages and Comparisons (v. 1, p. 123)
Insert instructions into computer spreadsheet cells to program arithmetic calculations. 12B/M6
Decide what degree of precision is adequate and round off the result of calculator operations to enough significant figures to reasonably reflect those of the inputs. 12B/M8
Convert quantities expressed in one unit of measurement into another unit of measurement when necessary to solve a real-world problem. 12B/M7b*
Ratios and Proportionality (v. 1, p. 119)
Numbers (p. 63)
Express numbers like 100, 1,000, and 1,000,000 as powers of ten. 12B/M9
Scale (p. 99)
Numbers (p. 65)
Estimate probabilities of outcomes in familiar situations on the basis of history or the number of possible outcomes. 12B/M10
Find the mean, median, and mode of a set of data. 12B/M4*
Mathematical Models (v. 1, p. 29)
Mathematical Processes (v. 1, p 27)
Statistical Reasoning (v. 1, p. 127)
Use, interpret, and compare numbers in several equivalent forms such as integers, fractions, decimals, and percents. 12B/M2
Statistical Reasoning (v. 1, p. 127)
Estimate distances and travel times from maps and the actual size of objects from scale drawings. 12B/M5
Use the units of the inputs to a calculation to determine what units (such as seconds, square inches, or dollars per tankful) should be used in expressing an answer. 12B/M7a*
Find what number is a given percentage of another number. 12B/M1b*
Ratios and Proportionality (v. 1, p. 119)
Computations (as on calculators) can give more digits than make sense or are useful. 9A/M7
Find what part one number is of another and express it as a fraction or a percentage. 12B/M1a*
3-5
Judge whether measurements and computations of quantities such as length, weight, or time are reasonable by comparing them to familiar values. 12B/E3
Make rough estimates of numerical calculations and use them to judge whether the results of a calculation done on a calculator are reasonable. 12B/E11** (BSL)
Ratios and Proportionality (v. 1, p. 119)
Numbers (p. 65)
Mathematical Processes (v. 1, p. 27)
Add and subtract fractions with common denominators. 12B/E10**
Use fractions and decimals, translating when necessary between commonly encountered fractions (halves, fourths, fifths, tenths, and hundredths) and their decimal equivalents. 12B/E2*
Make calculations when necessary to solve real-world problems and decide whether to make the calculation mentally, on paper, or with the help of a calculator or computer. 12B/E1*
State the purpose of each step in a calculation. 12B/E4
Mathematical Processes (v. 1, p 27)
Numbers (p. 65)
9A/E4
Mathematical Models (v. 1, p. 29)
It is possible (and often useful) to estimate quantities without determining them exactly. 9A/E7** (BSL)
Multiply any two whole numbers between 1 and 10 and multiply or divide any number by 2, 10, or 100. 12B/E7** (SFAA)
Add or subtract any two whole numbers between 1 and 100. 12B/E6** (SFAA)
Use a calculator to add, subtract, multiply, and divide any two whole or decimal numbers. 12B/E8** (SFAA)
Use appropriate units when describing quantities. 12B/E9**
9A/E3*
Ratios and Proportionality (v. 1, p. 119)
Numbers (p. 65)
Fractions are numbers used to represent part of something. 9A/E5** (SFAA)
K-2
Make quantitative estimates of time intervals and the lengths and weights of familiar objects. 12B/P5*
Explain to other students how they go about solving numerical problems. 12B/P4
Mathematical Models (v. 1, p. 29)
Give the sums and differences of single-digit numbers. 12B/P2*
Mathematical Processes (v. 1, p. 27)
9A/P1*
Using Tools and Devices (p. 109)
Graphic Representation (v. 1, p. 115)
Symbolic Representation (v. 1, p. 117)
Scale (p. 99)
Use whole numbers in ordering, counting, identifying, measuring, and describing objects and events. 12B/P1*
Mathematical Processes (v. 1, p. 27)
Ratios and Proportionality (v. 1, p. 119)
Numbers (p. 65)
Measure the length in whole units of objects using rulers and tape measures. 12C/P4*
estimation
computation